Math 43 Midterm 2 Review

In addition to the following review questions, you must be able to solve any of the questions from the 3D Lines & Planes handout.

Some questions below will require the use of a calculator.

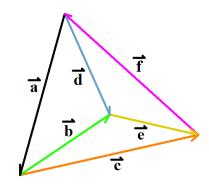
- [1] Consider the vectors $\vec{f} = 2\vec{j} 3\vec{k}$ and $\vec{g} = -\vec{i} 3\vec{j} + 4\vec{k}$.
 - [a] Find the angle between \vec{f} and \vec{g} . (Your answer should be in radians, rounded to 2 decimal places.)
 - [b] Find a unit vector perpendicular to both \vec{f} and \vec{g} . (Do <u>NOT</u> use decimal approximations.)
 - [c] If the terminal point of \vec{g} is (-7, 4, -8), find the initial point.
 - [d] If $\vec{h} = a\vec{i} + b\vec{j} 5\vec{k}$ is parallel to \vec{g} , find the values of \vec{a} and \vec{b} .
 - [e] If $\vec{e} = 7\vec{i} + c\vec{j} 5\vec{k}$ is perpendicular to \vec{g} , find the value of c.
- Let P be the point (-5, -2, 3). Let Q be the point (3, 2, -1). Let R be the point (-3, 4, -2).

Let \vec{u} be the vector with initial point R and terminal point Q.

Let \vec{w} be the vector with initial point P and terminal point R.

Let
$$\vec{t} = 3\vec{i} - \vec{k}$$
.

- [a] In which octant is R?
- [b] If you start at point P, move 2 units down, 4 units back, and 6 units to the right, find the co-ordinates of your ending point.
- [c] Write \vec{u} in component form.
- [d] Write \vec{w} as a linear combination of \vec{i} , \vec{j} and \vec{k} .
- [e] Find the magnitude of \vec{w} . (Do <u>NOT</u> use decimal approximations.)
- [f] Find a unit vector in the opposite direction as \vec{w} . (Do <u>NOT</u> use decimal approximations.)
- [g] Find a vector of magnitude 6 in the same direction as \vec{u} . (Do <u>NOT</u> use decimal approximations.)
- [h] If $\|\vec{v}\| = 3$, and the angle between \vec{u} and \vec{v} is 2 radians, find $\vec{u} \cdot \vec{v}$. (Round your answer to 2 decimal places.)
- [i] If $\|\vec{v}\| = 3$, and the angle between \vec{u} and \vec{v} is 2 radians, find the magnitude of $\vec{u} \times \vec{v}$. (Round your answer to 2 decimal places.)
- [j] Find the area of triangle PQR. (Do <u>NOT</u> use decimal approximations.)
- [k] Find $\angle QRP$. (Round your answer to 2 decimal places.)
- [1] Find the general equation of the plane passing through P, Q and R.
- [m] Find parametric equations for the line which passes through P and is also parallel to \vec{u} .
- [n] Find symmetric equations for the line which passes through Q and is also perpendicular to the plane -2x-3y+z=9.
- [o] Find the equation of the sphere with P and Q as endpoints of a diameter.
- [p] Find the volume of the parallelepiped with \vec{u} , \vec{w} and \vec{t} as adjacent edges.
- [3] Which octant or octants contain all points (x, y, z) where xz < 0?
- [4] Consider the sphere $x^2 + y^2 + z^2 4x + 6y + 10z + 29 = 0$.
 - [a] Find the center and radius of the sphere.
 - [a] Find the equations of the xy -, xz and yz traces of the sphere, and describe each trace.
- [5] Write vectors \vec{d} , \vec{e} and \vec{f} in terms of vectors \vec{a} , \vec{b} and \vec{c} in the diagram on the right.



Math 43 Midterm 2 Review Answers

[1] [a]
$$\approx 2.94$$
 radians

[b]
$$<-\frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}> or <\frac{1}{\sqrt{14}}, -\frac{3}{\sqrt{14}}, -\frac{2}{\sqrt{14}}>$$

[c]
$$(-6, 7, -12)$$

[d]
$$a = \frac{5}{4}, b = \frac{15}{4}$$

[e]
$$-9$$

[b]
$$(-9, 4, 1)$$

[c]
$$<6,-2,1>$$

[d]
$$2\vec{i} + 6\vec{j} - 5\vec{k}$$

[e]
$$\sqrt{65}$$

[f]
$$<-\frac{2}{\sqrt{65}}, -\frac{6}{\sqrt{65}}, \frac{5}{\sqrt{65}}>$$

[g]
$$<\frac{36}{\sqrt{41}}, -\frac{12}{\sqrt{41}}, \frac{6}{\sqrt{41}}>$$

[h]
$$\approx -7.99$$

[j]
$$2\sqrt{165}$$

[k]
$$\approx 1.47$$
 radians

[1]
$$x + 8y + 10z - 9 = 0$$

[m]
$$x = -5 + 6t$$
, $y = -2 - 2t$, $z = 3 + t$ OTHER ANSWERS POSSIBLE

[n]
$$\frac{x-3}{2} = \frac{y-2}{3} = -z - 1$$

OTHER ANSWERS POSSIBLE

[o]
$$(x+1)^2 + y^2 + (z-1)^2 = 24$$

[4] [a] center =
$$(2, -3, -5)$$
, radius = 3

[b] no
$$xy$$
 - trace

$$xz$$
 – trace is point $(2, 0, -5)$

$$yz$$
 – trace has equation $(y+3)^2 + (z+5)^2 = 5$ [circle in yz – plane, center = $(0, -3, -5)$, radius = $\sqrt{5}$]

[5]
$$\vec{d} = \vec{a} + \vec{b}$$
, $\vec{e} = \vec{c} - \vec{b}$, $\vec{f} = -\vec{a} - \vec{c}$